CHAPTER FIVE
RAPIDLY VARIED FLOW (RVF)

5.1 Characteristics of RVF

The flow is rapidly varied if the depth changes abruptly over a comparatively short distance; otherwise, it is called gradually varied flow. There is abrupt change of flow profile (virtually broken). Example: Hydraulic Jump, flow under gates

In view of contrast with GVF, the following characteristic features of RVF should be noted.

- Pronounced curvature ⇒ *hydrostatic pressure distribution* cannot be assumed
- Rapid variation in flow regime takes place in a very short distance.
- Effect of *boundary friction*, which would play a primary role in a GVF, is comparatively small and in most cases insignificant.
- In RVF the velocity-distribution coefficients α and β are much greater than unity and cannot be accurately determined.
- Flow is actually confined by separation zones as well as solid boundaries. (Because profiles could be broken).

5.2 Approach to the problem

The theory that assumes Parallel flow, Hydrostatic distribution of pressure does not apply in RVF computation. For RVF of continuous flow profile a mathematical equation of flow can be established, based on an in viscid and potential flow condition.

Approach to the solution of such equation include

- Graphical method (e.g. flow-net analysis)
- Numerical method (e.g. method of relaxation)

Despite such developments, no satisfactory general solution has yet been obtained

Practical Hydraulicians want to treat Various RVF phenomena as isolate cases each with its own semi empirical/ empirical treatment. In most cases the experimental results are used empirically. The physical concepts of the aspects of the flow will be interpreted qualitatively using energy principle, momentum principle, geometry plus sometimes dimensional analysis.

Three isolated cases of RVF are discussed here.

- Flow over spillway
- Hydraulic jump
- Flow under gate

5.3 Flows over Spillways

Spillway: is a structure over or through a dam for discharging flood flows; overflow channel; opening built into a dam or the side of a reservoir to release (to spill) excess floodwater.

5.3.1 Ogee-Crested overflow spillway

Crest shape

The ogee shaped crest is commonly used as a control weir for many types of spillways. The ogee shape which approximates the profile of the lower nappe of a sheet of water flowing over a sharp-crested weir provides the ideal form for obtaining optimum discharges. The shape of such a profile depends upon the head, the inclination of the upstream face of the flow section, and the height of the overflow section above the floor of the entrance channel (which influences the velocity of approach to the crest).

The ogee profile to be acceptable should provide maximum possible hydraulic efficiency, structural stability and economy and also avoid the formation of sub atmospheric pressures at the surface (which may induce cavitation).

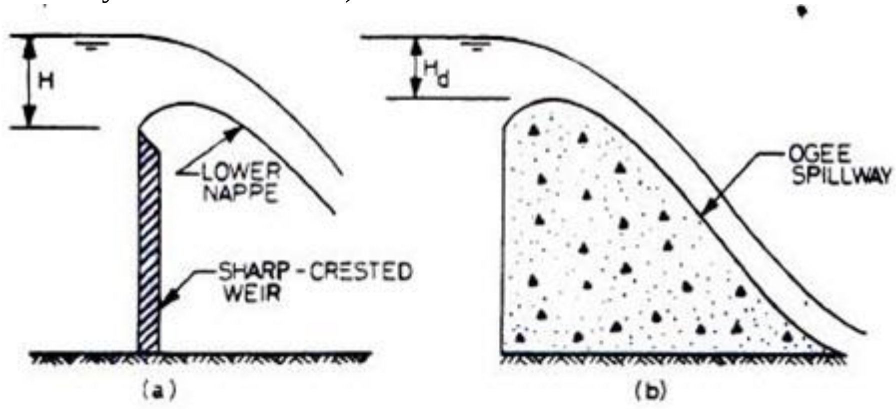


Figure 5.1 derivation of ogee spillway from sharp crested weir.

Ogee crested control structures are also sensitive to the upstream shape and hence, three types of ogee crests are commonly used

1. Ogee crests having vertical upstream face
2. Ogee crests having inclined upstream face
3. Ogee crests having over hang on upstream face

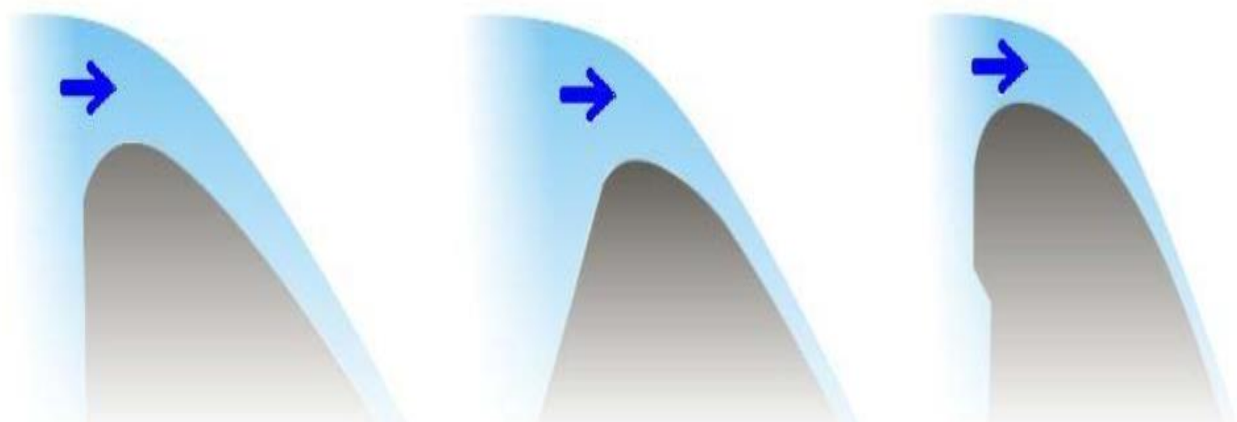


Figure 5.2 ogee crest weir with vertical, inclined and overhangs upstream face

The following equation as given by U.S. corps of engineers may be used for finding coordinates (X , Y) for the D/S profile

$$X^n = K H_d^{n-1} Y$$

Where:-

X and Y are Coordinates of the crest profile with the origin at the highest point of the crest.
Hd is the design head excluding the velocity head of the approach flow
K & n are parameters depending on the slope of the upstream face.

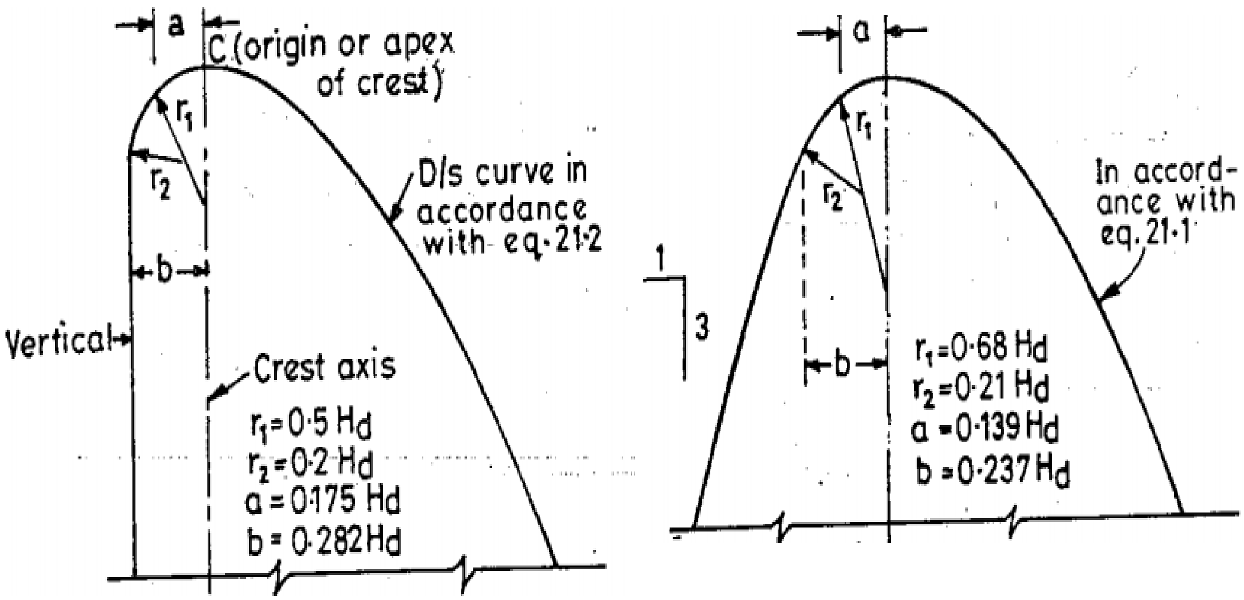


Figure 5.1 Overflow dams with different U/S faces

Table 5.1: Values of K, n, R_1, R_2, a, and b for different upstream faces

Shape of U/S face	K	N	R_1/H_d	R_2/H_d	a/H_d	B/H_d
Vertical	2.000	1.850	0.5	0.20	0.175	0.282
3V: 1H	1.936	1.836	0.68	0.21	0.139	0.237
3V: 2 H	1.939	1.810	0.48	0.22	0.115	0.240
3V: 3H	1.873	1.776	0.45	0.00	0.119	0.000

For intermediate slopes: approximate value of k and n may be obtained by plotting the above values against the corresponding slopes and interpolating from the plot the required values for any given slope within the plotted range.

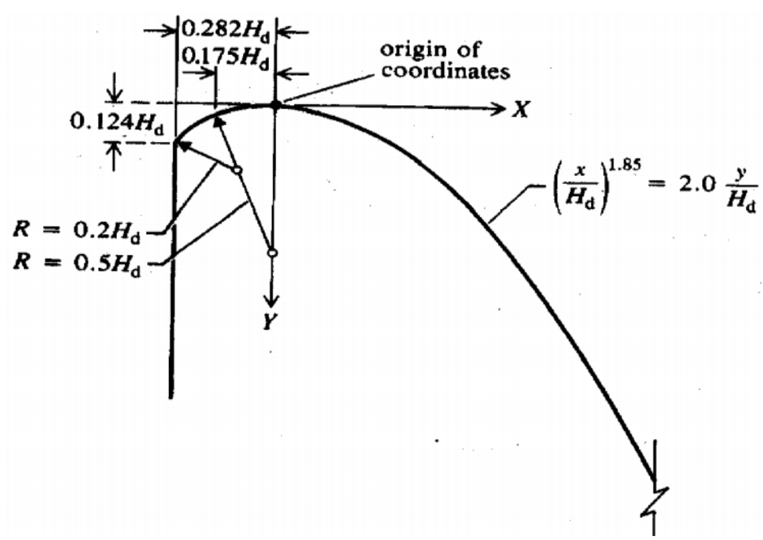


Figure 5.3 Overflow dams with vertical U/S face

The curved profile of the crest section is continued till it meets tangentially the straight sloping portion of the overflow dam section (spillway).The slope of the d/s face of the overflow dam usually varies in the range of 0.7(H):1(V) to 0.8:1 and is basically decided on the basis of stability requirements. The location of the point of tangent depends upon the slope of the d/s face, where the value of dy/dx for the curved profile and the straight segment must be equal at the end of the sloping surface of the spillway. At the end of the sloping surface a curved circular surface called BUCKET is provided to create a smooth transition of flow from spillway surface to river.

The BUCKET is also useful for dissipation of energy and prevention of scour.

Radius R of the bucket may be obtained approximately by the following empirical formula

$$R = 0.305 \times 10^{\frac{(V+}{\quad}}$$

Alternatively, – –

Where, P or h is spillway height

Hd= design head above the crest excluding velocity head

Velocity of flow may be approximated from the relationship (Neglecting the friction losses on the spillway surface.

$$\mathbf{V= [2g(Z+H_a-y)]^{1/2}}$$

Where, $\mathbf{Z= P+H_d}$ is the total fall from u/s water level to the floor level at the d/s toe

H_a = Head due to velocity of approach.

y = tail water depth

Alternatively USBR formula, $\mathbf{V= [2g(Z- 0.5\ H_d)]^{1/2}}$ can be used.

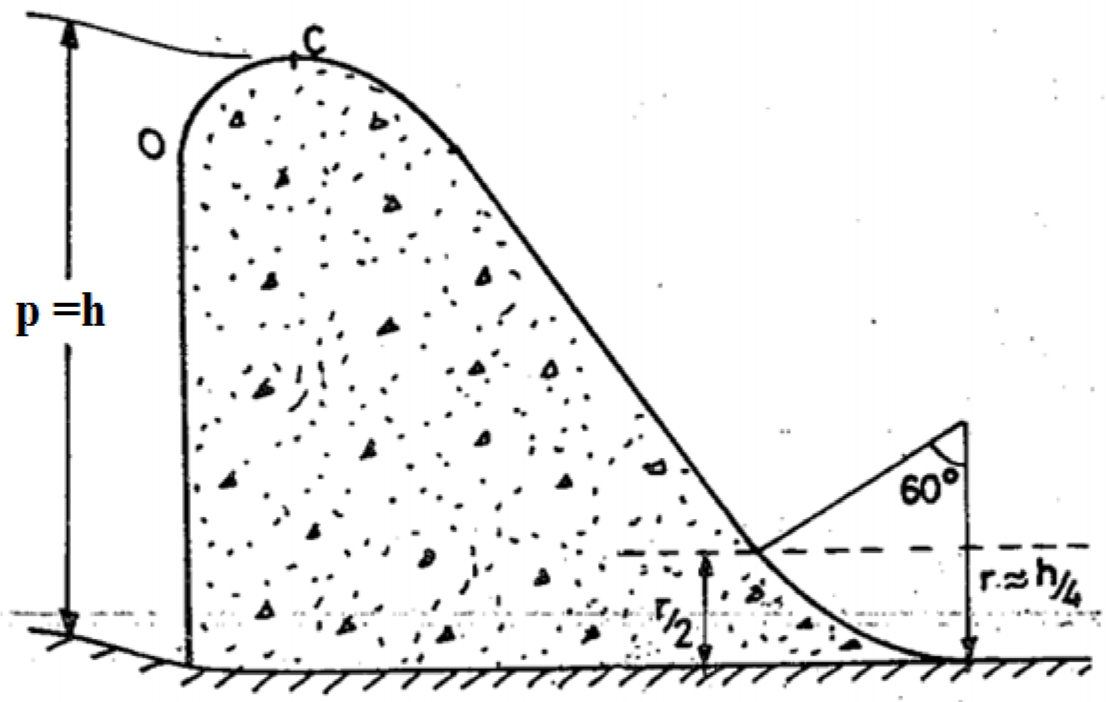


Figure 5.4 radius of bucket

U/S profile of the Weir Crest

Vertical U/S face: The u/s profile should be tangential to the vertical face and should have zero slope at the crest axis to ensure that there is no discontinuity along the surface of the flow. The u/s profile should conform to the following equation:

$$y = \{0.724\,(x + 0.270\,H_d)^{1.85}/(H_d)^{0.85}\} + 0.126\,H_d - 0.4315(H_d)^{0.375}(x + 0.270\,H_d)^{0.625}$$

Discharge characteristics of ogee crests-uncontrolled flow

For an ogee crested control weir for a spillway without any control with a gate, the free flow discharge equation is given as:-

$$Q = CLeHe^{1.5}$$

Where:-

- **L**e = effective length of overflow crest in meters
- **He** the total energy head on the crest, including the velocity head in the approach canal.
- The effect of the approach velocity is negligible when height **h** of the spillway is greater than 1.33Hd **(h > 1.33 Hd),** where **Hd** is the design head exclude the approach velocity head.
- Under this condition, i.e. h/Hd> 1.33, He = Hd can be taken (the approach velocity head is negligible) and
- the coefficient of discharge **C** has been found to be **C = 2.21** (if is in ft. C = 4.03)

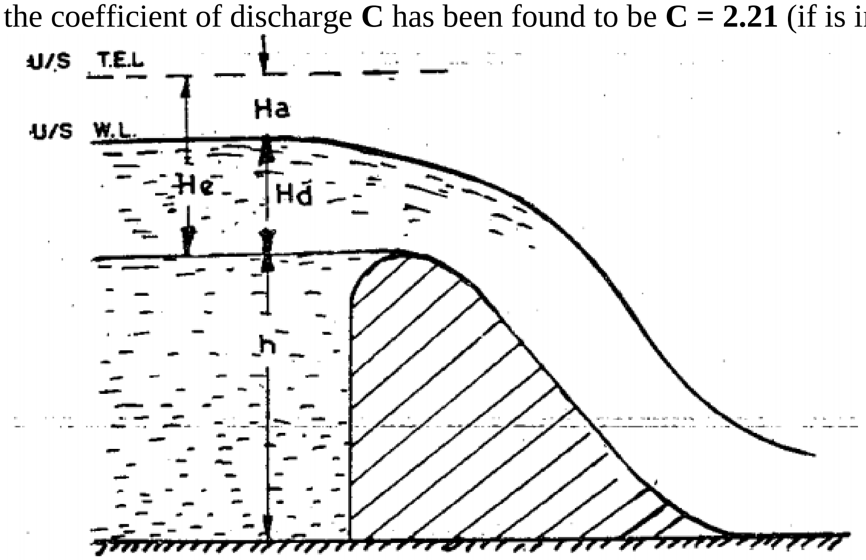


Figure 5.5 total energy head and design head on the crest

Where the crest piers and abutments are shaped to cause side contractions of the overflow, the effective length, Le, will be the net length of the crest, L. The effect of the end contractions may be taken into account by reducing the net length of crest as given below:

$$L_e = L - 2[N\,K_p + K_a]H_e$$

Where:-N is the number of piers and

Le is effective length of the crest

L is net length (clear water way)

K_p and K_a are the pier and abutment contraction coefficients.

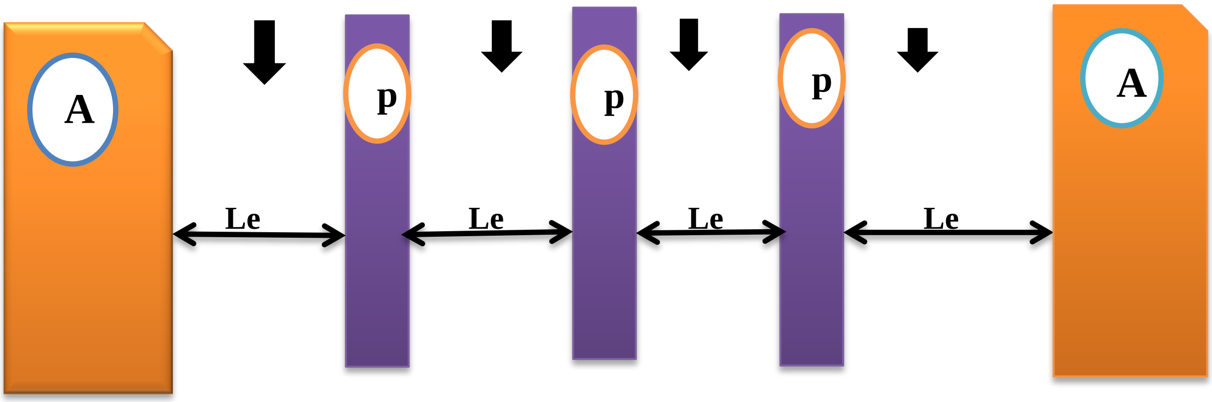


Figure 5.6 pier and abutment of ogee crested spillway

The pier contraction coefficient ***Kp*** depends upon the following factors:

- Shape and location of the pier nose
- Thickness of the pier
- Head in relation to the design head
- Approach velocity

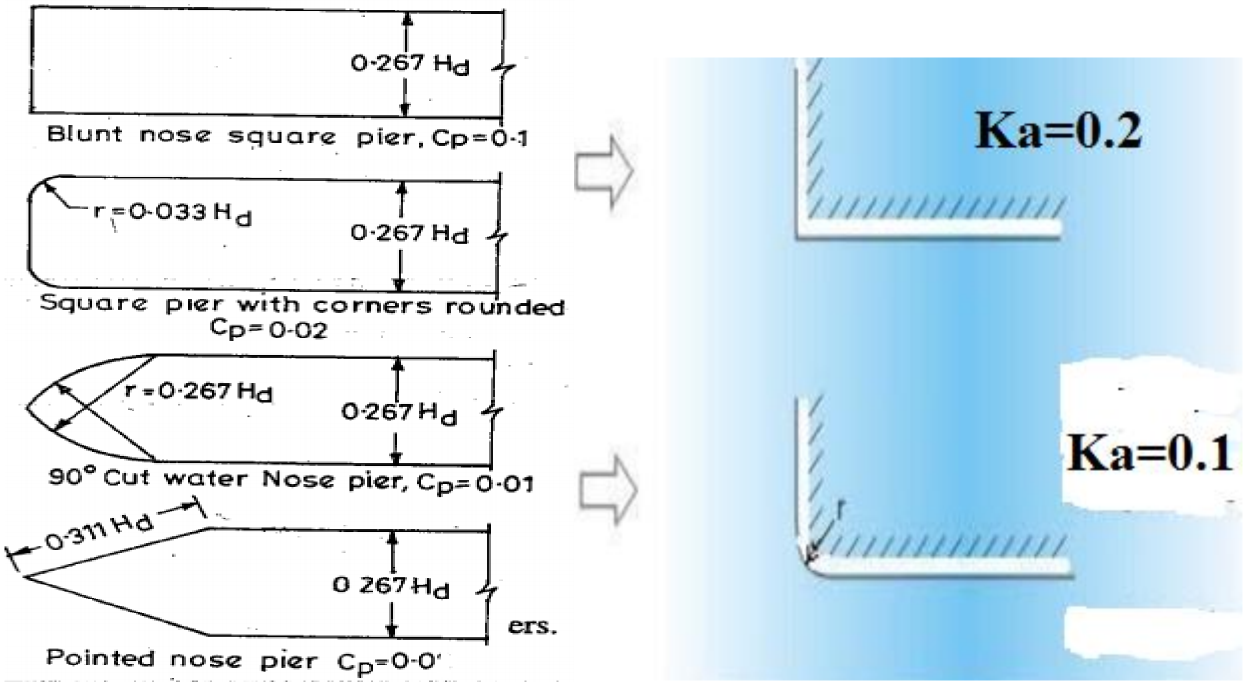


Figure 5.7 pier and abutment coefficients Kp and Ka

The abutment contraction coefficient is seen to depend upon the following factors:

- Shape of abutment
- Angle between upstream approach wall and the axis of flow
- Head, in relation to the design head
- Approach velocity

Table 5.1 values of Kp and Ka

S. No.	Pier Shape	Contraction coefficient K_p
1.	Square nosed piers without any rounding	0.1
2.	Square nosed piers with corners rounded on radius equal to 0.1 of pier thickness	0.02
3.	Rounded nose piers and 90° cut water nosed piers	0.01
4.	Pointed nose piers	0.0

S. No.	Shape of abutment	Contraction coefficient K_a
1.	Square abutment with head wall at 90° to the direction of flow	0.2
2.	Rounded abutment with head wall at 90° to the direction of flow	0.1

5.3.2 Broad-Crested and Sharp-Crested Weirs

Weirs are overflow structures that alter the flow so that:

1. Volumetric flow rate can be calculated,
2. Flooding can be prevented, or
3. Make a body of water more navigable
4. Increase the head of water

There are numerous types of weirs that have one or more of the functions listed above

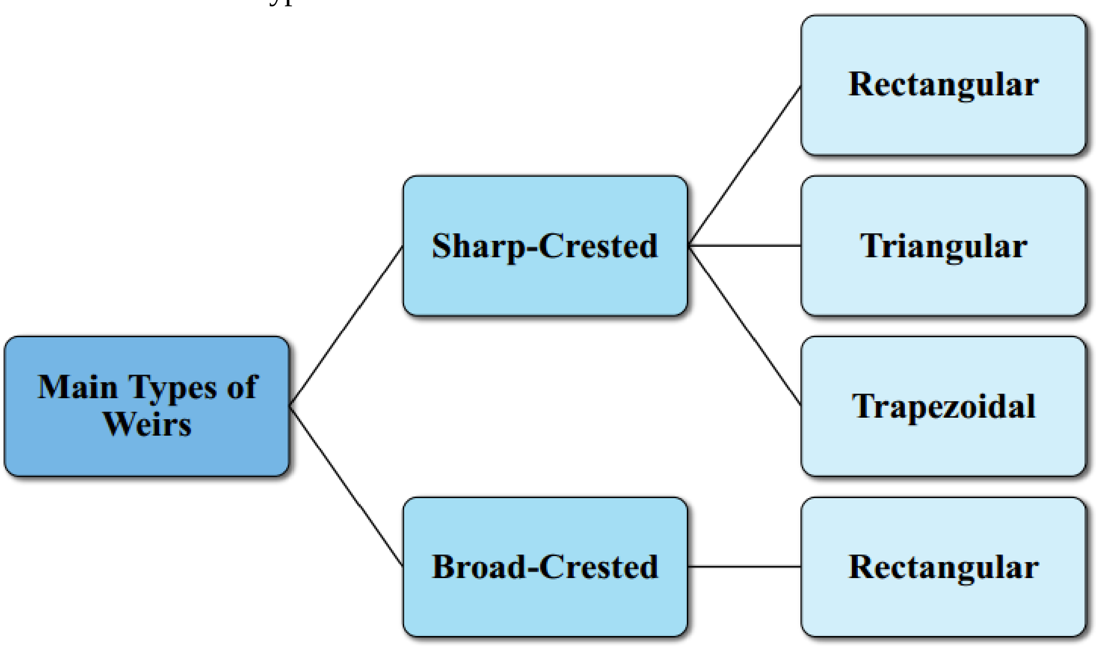


Figure 5.8 types of weir

SHARP-CRESTED WEIR

Critical depth (yc) occurs off the crest of the weir

Usually used to:

1. Measure the discharge of smaller rivers and canals
2. Change water elevation of smaller rivers and canals

BROAD-CRESTED WEIR

Critical depth (yc) occurs at the crest of the weir

Usually used to:

1. Measure the discharge of larger rivers and canals
2. Change water elevation of larger rivers and canal

5.3.2.1 BROAD-CRESTED WEIR(BCW)

Overflow structure with horizontal crest above which the deviation from a hydrostatic pressure distribution because of centripetal acceleration may be neglected.

Stream-lines are parallel and straight

Criteria:- $0.5 \geq H1/L \geq 0.07$

If $0.07 \geq H1/L$ the energy loss above the crest cannot be neglected

$0.5 \geq H1/L$ so that he hydrostatic pressure distribution can be assumed

Where: - L = length of the weir crest in the direction of flow,

H1 total energy head over the weir crest.

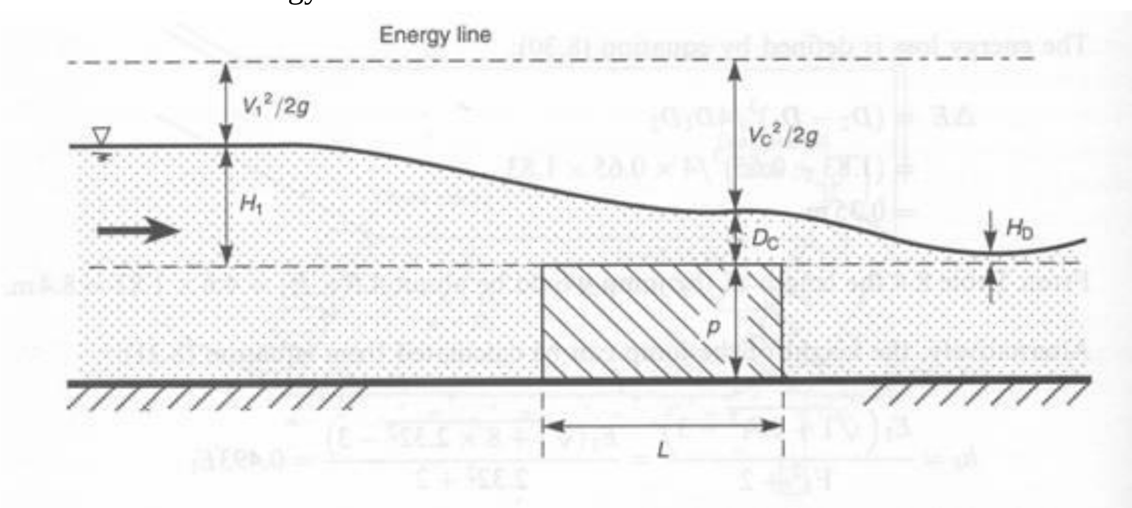


Figure 5.9 broad crested weir

Flow over a broad-crested weir is highly dependent on the weir's geometry

Simply discharge can be calculated as follows:

$$Q = C\,L\,H^{1.5}$$

Where:

Q = Volumetric flow rate

C = Constant for the specific weir structure (mostly ***C = 1.7***)

L = Width of the weir

H = is the measured head above the crest; excluding velocity head

5.3.2.2 Sharp Crested Weir (SCW)

Sharp crested weirs differ from broad crested weirs due to the detached water surface falling away from the downstream edge of the structure, known as a free-falling nappe The flow surfaces at the top and bottom of the nappe are exposed to the air and at atmospheric pressure The crest length in the direction of the flow is short enough not to influence the H-Q relationship of a weir. Overflow structure ($H_1/L > 15$). In practice, $0.002m \geq L$ so that even at a minimum head of 0.03m the nappe is completely free from the weir body after passing the weir. No adhered nappe can occur. An air pocket beneath the nappe form from which a quantity of air is removed continuously by the over falling jet.

Therefore, Precaution is required not to ensure that the pressure in the air pocket is not reduced. Otherwise resulting undesirable effects:

Owing to the increase of the under pressure the curvature of the over falling jet will increase, causing increase of the discharge coefficient Irregular supply of air to the pocket will cause vibration of the jet resulting an unsteady flow

SHARP CRESTED WIER is the simplest form of overflow spillway

Generally,

- Spillways must discharge the peak flow under smallest possible head.
- Negative pressure on the crest must be limited to avoid danger of cavitation on the crest or vibration of the structure.
- Theoretically, there should be atmospheric pressure on the crest

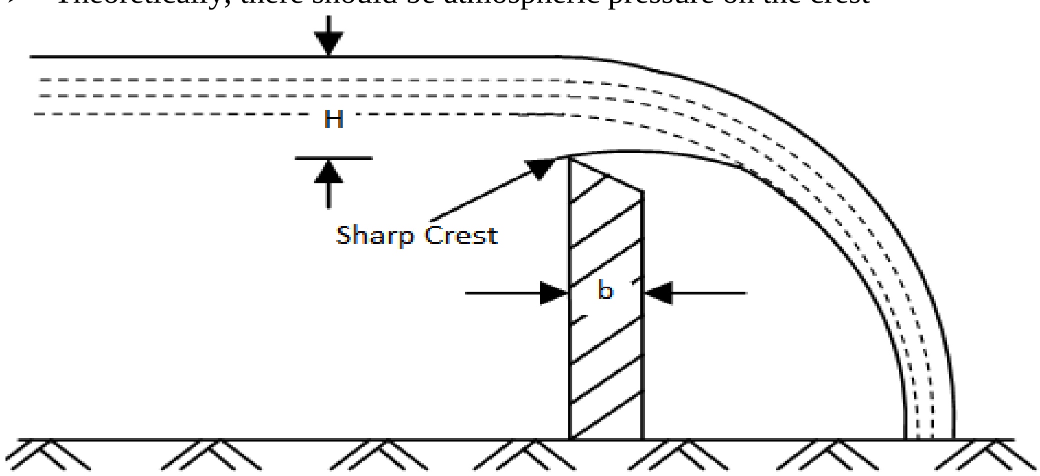


Figure 5.10 Sharp crusted weir

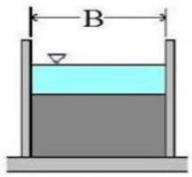
Design Consideration of Sharp-Crested Weir

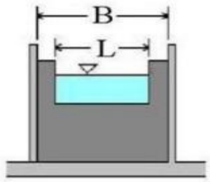
- The weir plate should be made of smooth metal free of rust and nicks
- When the plate thickness exceeds 1/8thinch, the downstream edge of the crest should be beveled to allow the nappe to detach from the weir
- When the width of the weir crest is equal to the width of the channel (suppressed shape), the air pocket under the nappe may become entrained and collapse, causing inaccurate flow calculations

1. Rectangular Sharp-Crested Weir

- Used to control water up- and downstream of weir
- Typically have higher discharge values
- Two main types:

1. Suppressed weir- crest is across the width of channel
2. Contracted weir - has notch cut into it, adding to the head loss





Rectangular and suppressed weirs have the same general discharge equation (below), but Differing weir lengths that the water flows over

$$Q = \frac{2}{3} C_D \sqrt{2g} B H^{3/2}$$

$$C_D = 0.602 + 0.083 \frac{H}{P}$$

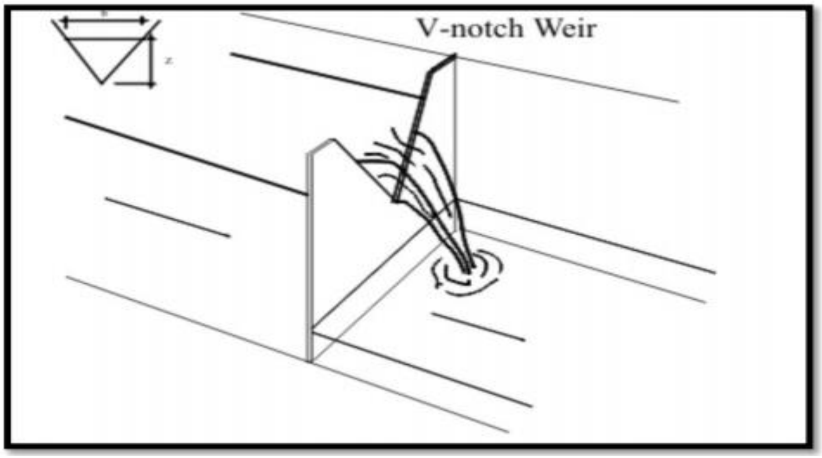
Where:

- Q (m^3/s) is the volumetric flow rate over the weir
- C_D is the discharge coefficient usually ranging from 0.60 to 0.62
- H (m) is the head over the weir(from the weir crest to the upstream water surface)
- P (m) is the height of the weir plate
- B (m) is the width of the contracted notch (rectangular), or the width of the channel (suppressed)
- g is the acceleration of gravity (9.81 m/s^2)

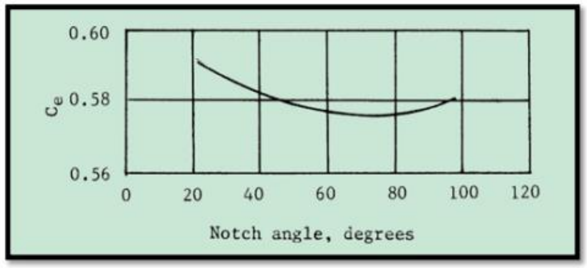
2. Triangular(V –Notch) Sharp-Crested Weir

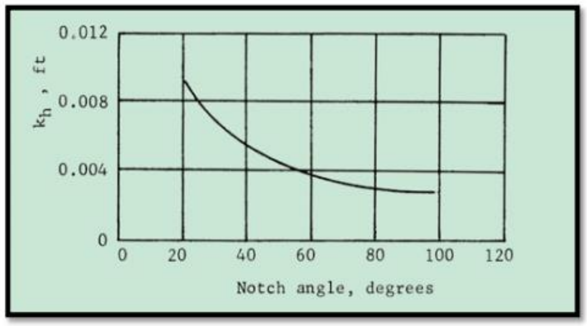
Used in cases of small discharge

- Best weir to measure discharge in an open channel
- Highest accuracy when measuring flow rate (usually +/- 2%)



- Calculating discharge across a V-Notch weir is more complicated:
  - $Q = \frac{8}{15}\sqrt{2g}C_e \tan\left(\frac{\theta}{2}\right) h_e^{5/2}$
  - $h_e = h_u + K_h$
- Where:
  - Q (m^3/s) is flow over V-Notch weir
  - C_e , K_h can be found using the graphs to the right
  - h_u (m) is the head flowing through the notch
  - θ (degrees) is the notch angle
  - g is the acceleration of gravity(9.81 m/s^2)
- When θ=90° this equation can be simplified to:
  - $Q = 2.49h_e^{2.48}$
  - for 0.2 ft < h_e < 1.25 ft

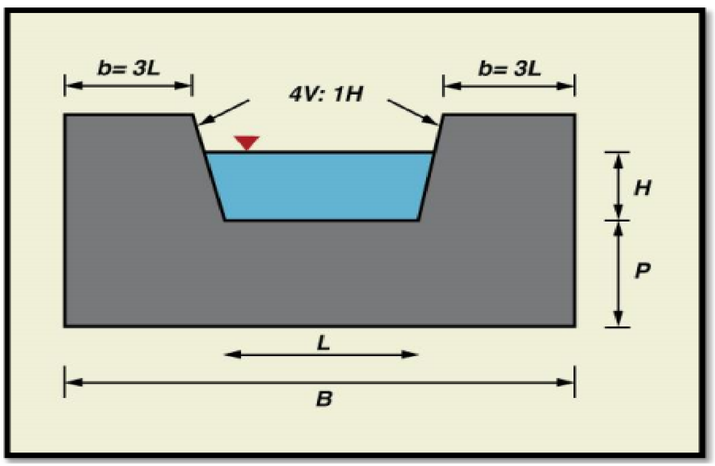




3. Trapezoidal Sharp-Crested Weir

These weirs are trapezoidal shaped with notch side slopes of **4:1** (***vertical: horizontal***)

- Combination of a rectangular and triangular weir
- These weirs are commonly used for irrigation
- Used when discharge is too great for a rectangular weir



Discharge for a trapezoidal Weir is calculated as follows:

$$\boldsymbol{Q = 3.367LH^{3/2}}$$

- Contractions in the free-flowing nappe occur in non-suppressed weirs because water travelling along the faces of the weir cannot instantaneously “turn” around the corners of the weir plate.
- A weir is fully contracted if B>4H and partially contracted if 0<B<4H
- The presence of contractions requires a discharge correction factor, but trapezoidal weirs are designed so that no correction is required

5.3.3 Aeration of the Nappe

In the preceding discussion the over falling nappe is considered aerated; i.e., the upper and lower nappe surfaces are subject to full atmospheric pressure. In practice, usually insufficient aeration below the nappe occurs due to removal of air by over falling jet.

Effects of reduction of pressure

- Increase in pressure difference on the spillway itself
- Change in the shape of the nappe for which the spillway crest is designed
- Increase in discharge, sometimes accompanied by fluctuation or pulsation of the nappe, which may be very objectionable if the weir or spillway is used for measuring purposes.
- unstable performance of the hydraulic model

5.4 Hydraulic Jump

The theory of jump developed is for horizontal or slightly inclined channels in which the weight of water in the jump has little effect upon the jump behavior and hence is ignored in the analyses. The results thus obtained however can be applied to most channels encountered in engineering problems.

For channels of large slope, the weight effect of water in the jump may become so pronounced that it must be included in the analysis.

1. Practical Applications

- Hydraulic jump is used to dissipate energy in water flowing over a dam, weir and other hydraulic structure and thus, prevent scouring d/s from the structure.
- To recover head or raise the water level on the d/s side of a measuring flume and thus maintains high water level in the channel for water distribution purposes.
- To increase weight on the apron and reduce uplift pressure by raising the water depth on the apron.
- To increase the discharge of a sluice gate by holding sack tail water, thus preventing drawn jump.
- To mix chemical used for water purification.
- To aerate water for city water supplies

2. Jump in Horizontal Rectangular channel

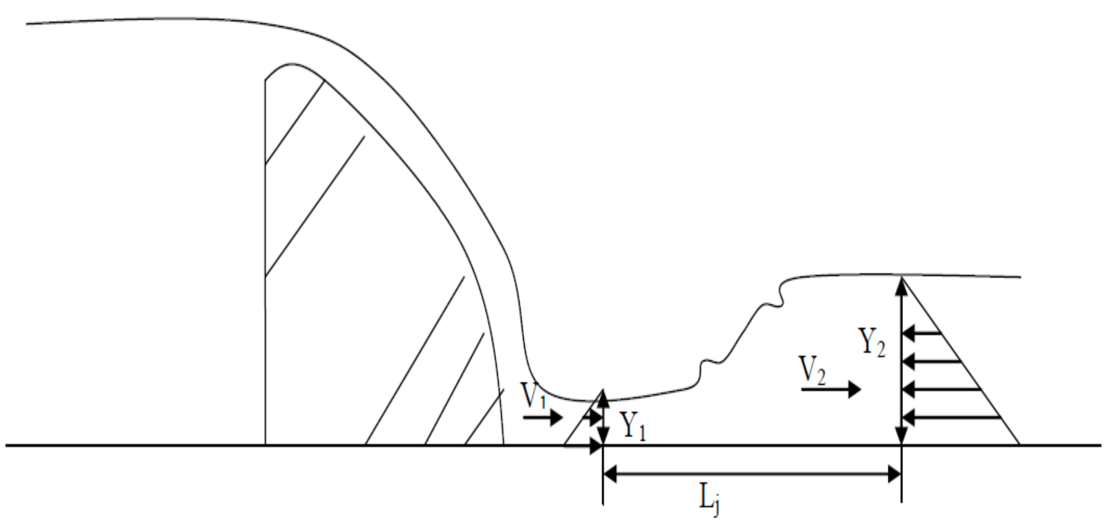


Figure 5.4: Hydraulic jump on horizontal bed following over a spillway

Where: V1 = velocity before jump
V2 = velocity after jump
y1 = water depth before jump
y2 = water depth after jump
Lj = length of jump

For supercritical flow in a horizontal rectangular channel, the energy of flow is dissipated through frictional resistance along the channel, resulting in a decrease in velocity and an increase in depth in the direction of flow.

A hydraulic jump will form in the channel if the Frond Number Fr1 of the flow, the flow depth y1, and a drown stream depth y2 satisfy the following equation:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_{r1}^{\ 2}} - 1\right)$$

This has been verified with experiments

3 .Types of Jump

Hydraulic Jumps on horizontal floor are of several distinct types. They can be conveniently classified according to Froude Number Fr1 of the incoming flow as follows.

1. **Critical jump (Fr1 =1):-** critical flow no jump can form
2. **Undular jump (1< Fr1< 1.7):-**the water surface shows undulation
3. **Weak jump (1.7 < Fr1< 2.5):-** A series of small rollers develop on the surface of the jump, but the d/s water surface remains smooth. The velocity throughout is fairly uniform, and the energy loss low.
4. **Oscillating Jump (2.5 < Fr1< 4.5):-** there is an oscillating jet entering the jump bottom to surface and back again with no periodicity. Each oscillation produces a large wave of irregular period which, very common in canals, can travel for miles doing unlimited damage to earth banks and ripraps.
5. **Steady Jump (4.5 < Fr1< 9.0): -** The downstream extremity of the surface roller and the point at which the high-velocity jet tends to leave the flow occur at practically the same vertical section. The action and position of this jump are least sensitive to variation in tail-water depth. The jump is well balanced and the performance is at its best. The energy dissipation ranges from 45 to 70%.
6. **Strong jump (Fr> 9.0): -** The high-velocity jet grabs intermittent slugs of water rolling down the front face of the jump, generating wave‟s down-stream and a rough surface can prevail. The jump action is rough but effective since the energy dissipation may reach 85%.

N.B: It should be noted that the ranges of the Froude Number given above for the various types of jump are not clear-cut but overlap to a certain extent depending on local conditions.

4. Basic Characteristics of the Jump

Energy Loss: the loss of energy in the jump is equal to the difference in specific energy before and after the jump.

$$\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\text{Relative loss : the ratio } \frac{\Delta E}{E_1}$$

Efficiency: the ratio of the specific energy after the jump to that before the jump is defined as the efficiency of the jump.

$$\frac{E_2}{E_1} = \frac{(8F_1^2 + 1)^{3/2} - 4F_1^2 + 1}{8F_1^2 (2 + F_1^2)}$$

This equation indicates that the efficiency of a jump is a dimension less function, depending only on the Froude Number of the approach flow. The relative loss is equal to

$$1 - \frac{E_2}{E_1}$$; this also is a dimensionless function.

Height of Jump: - the difference between the depths after and before the jump.

Hj = y2 – y1

Expressing the above term as a ratio with respect to initial specific energy.

$$\frac{h_1}{E_1} = \frac{y_2}{E_1} - \frac{y_1}{E_1}$$

,

Where $\frac{hj}{E1}$ is the relative height, $\frac{y_1}{E_1}$ is the relative initial depth, and $\frac{y2}{E1}$ is the relative Sequence depth. All these ratios can be shown to be dimensionless function of F1. For example

$$\frac{h_j}{E_1} = \frac{\sqrt{1+8F_1^2} - 3}{F_1^2 + 2}$$

Length of Jump:-The length of a jump (also length of stilling basin) is empirically given as

$$L = k(y_2 - y1)$$

Where, k – is a coefficient derived from laboratory and filed experiment. 4.5 < k < 5.5

Where the lower k = 4.5 applies of Fr2> 10 and the highs for Fr2< 3.

5.5 Flows under Gates

Gates in canals are mainly used as water level regulators. Sometimes, gates are used as discharge regulator (measuring device). They are under-shot or underflow structures. Example slice gate, radial gate roller gate.

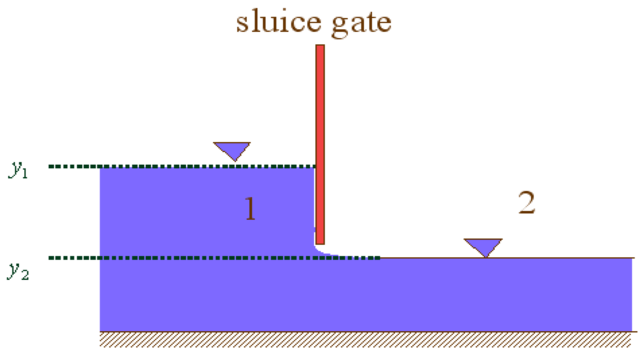


Figure 5.5 flow under sluice gate

The design of underflow gate focuses on head-discharge relationship (Q-H). The objective is to minimize head loss; this means that the gate has to be lifted out off the water for design discharge. The other concern of the design is the pressure distribution over the gate as a function of opening and gate form.

The H-Q relationship for gate depends on the shape and dimension of the control section and the resulting curvature of the streamlines. For gated structures the control section is defined by the vena contract, being the smallest cross section just down steam of the gate. In the vena contract, streamlines are straight and parallel.

In gate flow 3 flow types can be distinguished.

1. **Free flow:** - the opening is relatively small $\left(\frac{1}{\,} > \boldsymbol{a}\right)$ and the contraction of the steam-lines in vertical direction is strong. The downstream water level (h_2) won't affect the flow underneath the gate and a hydraulics jump will occur downstream of the vena contra. The discharge depends up on the gate opening the upstream water level and the contraction coefficient.
2. **Submerged flow**: - the d/s water level influences the flow underneath the gate. The hydraulic jump is drowned and the jet underneath the gate is submerged. The discharge depends upon the upstream and downstream water level and the gate opening.

The boundary between free and submerged flow is a sharp one, which can be cleanly found from the gate opening and the two water levels.

3. **Weir flow**:-on off gate

The Discharge equation for a free flow underneath a sharp edged gate is:

$$Q = C_d Ba\sqrt{2gh_1}$$

Where: Cd= discharge coefficient

B = Width of gate opening

a= height of gate opening

h1= upstream water depth

The discharge coefficient Cd is given by

$$C_d = \frac{C_c}{\sqrt{1 + C_c \frac{a}{h_1}}}$$

Where, CC = Contraction coefficient of the jet depending on the shape of the gate and on —

d = diameter of the rounded bottom edge

$$\text{For } For\frac{d}{a}<4.7 : C_C = C_c = 0.51 + 0.1*\sqrt{23.04-\left(\frac{d}{a}-4.69\right)^2}$$

$$\text{For } \frac{d}{a}\geq 4.7 ; C_c = 0.99 \ (Rounded \ edged \ gates)$$

Where d is diameter of the rounded bottom edge.
For sharp edged gates d is small and CC = 0.61.
The limit between tree flow and submerged flows follows from.

$$\frac{h2}{a} = \frac{C_C}{2} + \left[1+16\left(\frac{H_1}{aC_C}-1\right)-1\right]$$

Where:
h2= downstream water level
H1= upstream energy level
CC is 0.611 for sharp edged gates (d=0) and CC is 0.99 for rounded of edged gates with –

For submerged flow, some equations include the difference between the upstream and downstream depths and others use the upstream water level only. The general equation is given as.

$$Q = C_2 Ba \sqrt{2gh_1}$$

Where,
a = vertical opening of the gate (a< 0.67h1)
h1= Upstream water depth
B = Effective width of the opening
C2= discharge coefficient.
The equation is the same as for free flow but the discharge coefficient C2 is a function of $\frac{h1}{a}, \frac{h2}{a}$ $and C_C$,
Where h2 is downstream water depth and C2 values range between 0 and 1.
Others roughly classify the flows as,

$$\frac{h_1}{a} > 2 \ - free \ flow$$

$$1.5 < \frac{h_1}{a} < 2 - submerged$$

$$\frac{h_1}{a} < 1.5 \ - weir \ flow$$

For values of $a > 0.67*h_1 \ or 1 > 0.67\frac{h_1}{a} \Rightarrow \frac{h_1}{a} < 1.5$ the discharge follows the equation for a broad-crested weir.

$$Q = 1.7*B*H^{3/2}$$